Relativity and Quantum Mechanics — Conflict or Peaceful Coexistence

first page

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Introduction

Nonlocality in quantum mechanics provides a prima facie conflict with the foundations of special relativity which is usually taken to forbid causal processes operating outside the light cone, i.e. at space-like separation.

In the recent literature various senses of locality have been distinguished and the question of which of these is violated by quantum mechanics depends crucially on the interpretation of quantum mechanics adopted.

In the case of deterministic hidden-variable interpretations action-at-a-distance, in the sense of instantaneously affecting the processed value of some attribute belonging to one member of a pair of ~~physically~~ spatially separated systems by altering the physical arrangement of an apparatus

interacting with the other member appears to be clearly demonstrable. Here a direct conflict with special relativity seems difficult to avoid except by appealing to the problematic topic of faster-than-light interactions.[2]

In the case of indeterministic or stochastic hidden-variable interpretations the issues raised are much more subtle. No straightforward derivation of the Bell inequalities along the lines available in the deterministic case is possible. Appeal has to be made to two conditions spelled out in detail by Jarrett (1984).

The first condition, Jarrett locality (or, as Shimony prefers to call it parameter independence[3]) is equivalent to requiring that one cannot use spacelike-separated mechanical correlations for signalling.[4]

The second condition, Jarrett completeness (or in Shimony's terminology outcome independence) identifies the hidden-variable description of the source as the common cause of the processed values of spin-components in the idealized Bohm-Bell experiment.

If Jarrett completeness is violated it is often argued that the spatially separated

stochastic causal link on the grounds that the correlations can only be accounted for on the basis of ~~a common cause or~~ stochastic links to a common cause or a direct stochastic causal link. The purpose of the present paper is to question this conclusion by exhibiting a 'third way' which may arguably be regarded as a mathematical explication of what Shimony ~~in his (1984)~~ has termed passion-at-a-distance.[4]

Necessary conditions for stochastic causality

Causal relations exhibit a quality of robustness. Suitably small disturbances of either relata do not modify or affect the causal relation. This is essentially the basis of the mark method for identifying causal processes.[5,6,7] The processes propagate small disturbances (marks) in a local event-structure in accordance with the causal law at issue.

We shall translate this idea in the context of stochastic causal relations by ~~exhibiting~~ formulating

~~5. For the mark method see Reichenbach (1928) and (1956) and Salmon (1984)~~ discussion of

the following necessary condition for stochastic causality:

Physical magnitudes $\underline{a}$ and $\underline{b}$ pertaining to two two separated regions A and B exhibit a stochastic causal connection only if there exists a class of disturbances acting on B (A) such that $\underline{b}$ ($\underline{a}$) becomes ill ($\underline{b}$) from these disturbances.

Denoting the disturbance acting on B by d and the possible values of d by ε_a and of $\underline{b}$ by ε_b then the first part of this condition can be rendered formally as

$$\exists D \left(\forall d \in D \left(\text{Prob}(\underline{a} = \varepsilon_a / \underline{b} = \varepsilon_b \,\&\, d) \right.\right.$$
$$\left.\left. = \text{Prob}(a = \varepsilon_a / b = \varepsilon_b) \right) \right) \quad (1)$$

A similar condition can be written down for disturbances acting on A.

We are now going to apply this condition to the case of the singlet state $|\Psi\rangle$ of two distinguishable spin-$\frac{1}{2}$ particles denoted by A and B. We let $\underline{a}$ denote the spin-projection $\sigma_A \cdot \hat{a}$ along the direction of the unit vector $\hat{a}$. Similarly $\underline{b}$ denotes $\sigma_B \cdot \hat{b}$ for some, in general different, direction $\hat{b}$.

Then it is well-known that $\varepsilon_a = \pm 1$, $\varepsilon_b = \pm 1$.

~~of the particle B~~. Let ψ' be a variable ranging over these perturbed states. Then a necessary condition that $\underline{a}$ and $\underline{b}$ exhibit stochastic causal connection for arbitrary choice of the directions $\hat{a}$ and $\hat{b}$ is:

$$\forall \underline{a}\ \forall \underline{b}\ \exists D \left(\forall \psi' \in D \left(\mathrm{Prob}^{|\psi'\rangle}(\underline{a} = \varepsilon_a | \underline{b} = \varepsilon_b) = \mathrm{Prob}^{|\psi\rangle}(\underline{a} = \varepsilon_a | \underline{b} = \varepsilon_b) \right) \right) \quad \text{--- (2)}$$

We shall now show by explicit calculation that condition (2) is violated. Since (2) is a necessary condition for ~~a~~ a direct stochastic causal link between $\underline{a}$ and $\underline{b}$ it will follow that no such link exists.

B. The non robustness of the Singlet State

Denote the spin-projections of $\underline{\sigma}_A$ and $\underline{\sigma}_B$ along the arbitrarily chosen z-axis by σ_{Az} and σ_{Bz} respectively.

Then we have

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\sigma_{Az} = +1\rangle |\sigma_{Bz} = -1\rangle \right.$$
$$\left. |\sigma_{Az} = -1\rangle |\sigma_{Bz} = +1\rangle \right).$$

tensor product of the Hilbert spaces $\mathcal{H}_A$ and $\mathcal{H}_B$ for the particles A and B.

Consider an arbitrary perturbation acting on $\mathcal{H}_B$. It will induce a 2+2 unitary transformation on all modes belonging to $\mathcal{H}_B$. The transformation is thus an element of $U(2)$ the group of 2-dimensional unitary transformations.

But it is well known that $U(2)$ can be embedded as the direct product of $U(1)$ the group of 1-dimensional unitary transformations and $SU(2)$ the group of 2-dimensional unitary unimodular transformations. Formally

$$U(2) = U(1) \times SU(2) \quad (4)$$

Now an element of $U(1)$ merely induces a phase-shift which does not change the physical state (ray) associated with particle B and can be ignored in computing all probabilities. The action of an element of $SU(2)$ can always be represented as $e^{i(\sigma_B \cdot \hat{n})\phi/2}$ where the direction of the unit vector $\hat{n}$ and the magnitude of the angle ϕ range over the three parameters of the group (note that $0 \leq \phi \leq 4\pi$).

We denote $e^{i(\sigma_B \cdot \hat{n})\phi/2}$ by $U(\hat{n}, \phi)$

$$|\psi'\rangle = \frac{+}{\sqrt{2}}\left(|\sigma_{A2}=+1\rangle\, u(\vec{n},\phi)\,|\sigma_{B2}=-1\rangle - |\sigma_{A2}=-1\rangle\, u(\vec{n},\phi)\,|\sigma_{B2}=+1\rangle\right) \quad ---(5)$$

On what follows we shall consider the particular choice $\xi_a = \xi_2 = 1$

then we have the familiar results.

$$Prob^{|\psi'\rangle}(a=1) = 1/2 \quad ---(6)$$

$$Prob^{|\psi'\rangle}(2=1) = 1/2 \quad ---(7)$$

$$Prob^{|\psi'\rangle}(a=1\,|\,2=1) = \sin^2\tfrac{1}{2}\theta_{a2} \quad --(8)$$

where θ_{a2} is the angle between the directions $\hat{a}$ and $\hat{2}$

We are interested now in calculating $Prob^{|\psi'\rangle}(a=1)$, $Prob^{|\psi'\rangle}(2=1)$ and $Prob^{|\psi'\rangle}(a=1\,|\,2=1)$.

The robustness condition for Stuckelberg causality is simply

$$Prob^{|\psi'\rangle}(a=1\,|\,2=1) = Prob^{|\psi\rangle}(a=1\,|\,2=1) \quad ---(9)$$

We shall show that for <u>any</u> given detected state $|\psi\rangle$ there always

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* corresponding to an (active) clockwise rotation about the direction $\hat{n}$.

new hand

[To calculate the new probabilities apply the unitary transformation $U(\hat{n},-\phi)$ $(= U^{-1}(\hat{n},\phi))$ to the space H_B. This converts ~~$|\psi'\rangle$~~ $|\psi'\rangle$ back into $|\psi\rangle$, but induces a rotation $R(\hat{n},-\phi)$ in the operator $\underset{\sim}{\sigma}_B$. $R(\hat{n},\phi)$ is an element of the 3-dimensional rotation group $SO(3)$*. The above result is a direct expression of the famous homomorphism that exists between $SU(2)$ and $SO(3)$ ↑. Succinctly

$$U(\hat{n},\phi)\,\underset{\sim}{\sigma}_B\,U(\hat{n},-\phi) = R(\hat{n},\phi)\,\underset{\sim}{\sigma}_B \qquad (10)$$

On the left of Eq(10) U acts on the spinor indices of σ_B, while on the right R acts on the vector indices.

Thus the effect of our unitary transformation on the operators $\underline{a}$ and $\underline{b}$ is as follows.

$$\underline{a} = \underset{\sim}{\sigma}_A \cdot \hat{\underset{\sim}{a}} \longrightarrow \underline{a}' = \underline{a} \qquad (11)$$

$$\underline{b} = \underset{\sim}{\sigma}_B \cdot \hat{\underset{\sim}{b}} \longrightarrow \underline{b}' = \left(U(\hat{n},-\phi)\,\underset{\sim}{\sigma}_B\,U(\hat{n},\phi)\right)\cdot\hat{\underset{\sim}{b}}$$

$$= \left(R(\hat{n},-\phi)\,\underset{\sim}{\sigma}_B\right)\cdot\hat{\underset{\sim}{b}}$$

$$= \underset{\sim}{\sigma}_B \cdot \left(R(\hat{n},\phi)\,\hat{\underset{\sim}{b}}\right) = \underset{\sim}{\sigma}_B \cdot \hat{\underset{\sim}{b}}' \qquad (12)$$

where

$$\hat{\underset{\sim}{b}}' = R(\hat{n},\phi)\,\hat{\underset{\sim}{b}} \qquad (13)$$

Hence we have at once the following results:

(1) $\mathrm{Prob}^{|\psi'\rangle}(a=1) = \mathrm{Prob}^{|\psi\rangle}(a'=1) = \mathrm{Prob}^{|\psi\rangle}(a=1)$
$= 1/2$. — — (14)

This shows that the perturbation acting on particle B cannot be used to send signals to the location of particle A. Notice that this result is quite general, since it does not depend on the connection between $U(\vec{n},\theta)$ and $R(\vec{n},\theta)$, only on the existence of the unitary transformation U. In the appendix to this paper we shall show how our method of proof of this no-signalling theorem can be given in full generality.

(2) $\mathrm{Prob}^{|\psi'\rangle}(b=1) = \mathrm{Prob}^{|\psi\rangle}(b'=1) = \frac{1}{2}$ — — (15)

~~This follows from the rotational invariance of the singlet state (we can then always rotate the z-axis to be lie along the direction $\vec{b}'$)~~

This a rather surprising result that is a special property of the example under discussion.

(3) $\mathrm{Prob}^{|\psi'\rangle}(a=1|b=1) = \mathrm{Prob}^{|\psi\rangle}(a'=1|b'=1)$
$= \mathrm{Prob}^{|\psi\rangle}(a=1|b'=1) = \sin^2\frac{1}{2}\theta_{ab'}$ — — (16)

i.e. $\sin^2\frac{\theta_{n2}}{2} = \sin^2\frac{\theta_{n2'}}{2}$

or $\theta_{n2} = \theta_{n2'}$ — — — (17)

since the angles all lie in the range 0 to π.

(17) can be used to simple geometrical interpretation.

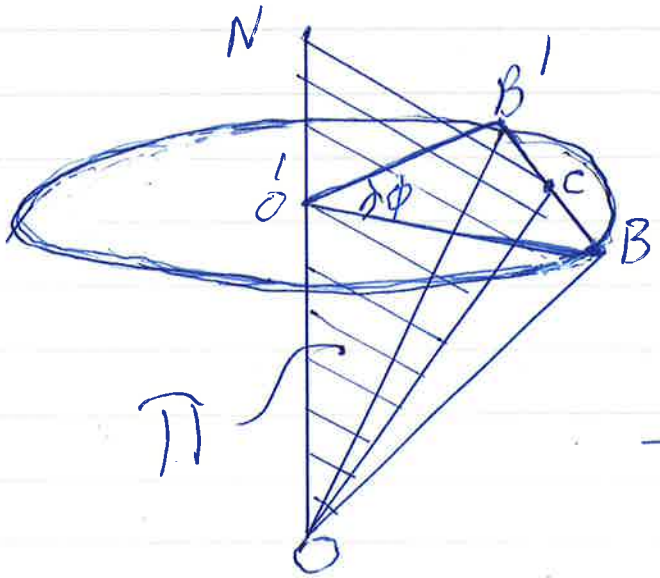

FIGURE 7.

In ~~the figure~~ FIGURE 7,
ON represents the unit vector $\hat{n}$
OB, — — — — — — $\hat{r}$
OB' — — — — — — $\hat{r}'$

B and B' lie on a circle with centre O' whose plane is perpendicular to ON.
$\angle BO'B' = \phi$.

Now $\theta_{n2} = \theta_{n2'}$ is equivalent to

$\cos\theta_{n2} = \cos\theta_{n2'}$

$\hat{n}\cdot\hat{r} = \hat{n}\cdot\hat{r}'$

Hence $\hat{a}$ must lie in the plane perpendicular to the line BB'. Call this plane Π. Π can equally well be characterized as the plane through ON and OC where OC is the bisector of the angle between $\hat{v}$ and $\hat{v}'$. The plane Π is shown shaded in the diagram.

So we have the following Theorem:

For any given perturbation on particle B, that issues in a rotation of $\hat{v}$ to $\hat{v}'$, the conditional probability $\text{Prob}^{|\psi\rangle}(a=1|b=1)$ will be invariant under the perturbation if and only if the direction $\hat{a}$ lies in the plane defined by the axis of rotation and the bisector of the directions $\hat{v}$ and $\hat{v}'$.

display (robust)

Corollaries

(1) If $\hat{n}$ coincides with $\hat{a}$
$\text{Prob}^{|\psi\rangle}(a=1|b=1)$ is robust for all $\hat{v}$.

(2) If $\hat{n}$ coincides with $\hat{v}$
$\text{Prob}^{|\psi\rangle}(a=1|b=1)$ is robust for all $\hat{a}$.

(3) For any perturbation (rotation) however small there always exist directions $\hat{a}$ and $\hat{v}$ for which $\text{Prob}^{|\psi\rangle}(a=1|b=1)$

It is this last corollary which demonstrates, I believe, that a and b cannot be regarded as related by stochastic causality. The correlations between a and b are a property of the particular quantum-mechanical state, viz. the singlet state, in which the particles emerge from the source. This property, it is claimed, explicates Shimony's passion-at-a-distance, and points the way to peaceful coexistence between quantum-mechanical nonlocality and special relativity.

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Appendix

The General No-Signalling Theorem

General previous proofs that quantum-mechanical correlations cannot be used for signalling have been given in the literature. 8, 9, 10, 4. See, in particular, Eberhard (1980), Ghirardi, Rimini and Weber (1980), Page (1982), Jordan (1983) and Shimony (1984). The following proof appears to combine generality with brevity.

Consider two systems $\underline{A}$ and $\underline{B}$ with associated Hilbert spaces H_A and H_B. $\underline{a}$ is any observable on H_A, extended to $\underline{a} \otimes \underline{1}$ on $H_A \otimes H_B$. Similarly $\underline{u}$ is any observable on H_B, extended to $\underline{1} \otimes \underline{u}$ on $H_A \otimes H_B$.

Consider any state $|\Psi(t)\rangle$ of the joint system at time t and denote by

$$\mathrm{Prob}(\lambda)^{|\Psi(t)\rangle}_{\underline{a}(t)\otimes 1}$$

the probability that the time-dependent observable $\underline{a}(t)\otimes 1$ has will yield the measurement result λ in the state $|\Psi_t\rangle$.

We shall work in the interaction representation so in the absence of perturbation $|\Psi(t)\rangle$ is constant, whilst in all cases $\underline{a}(t)\otimes 1$ evolves in time according to the unperturbed Hamiltonian.

Now perturb the system B in any

Then at time t', the state of the joint system is

$$|\Psi(t')\rangle = \left(\underline{I} \otimes U_B(t,t')\right)|\Psi(t)\rangle$$

We now compute

$$\text{Prob}(\lambda)^{|\Psi(t')\rangle}_{\underline{a}(t')\otimes I}$$

$$= \text{Prob}(\lambda)^{|\Psi(t)\rangle}_{\underline{a}'(t')\otimes I}$$

where $\underline{a}'(t')\otimes I = \left(I \otimes U_B^{-1}\right)\left(\underline{a}(t')\otimes I\right)\left(I \otimes U_B\right)$

$$= \left(I \otimes U_B^{-1}\right)\left(\underline{a}(t') \otimes U_B\right)$$

$$= \underline{a}(t') \otimes \left(U_B^{-1} U_B\right) = \underline{a}(t') \otimes I.$$

$$\therefore \quad \text{Prob}(\lambda)^{|\Psi(t')\rangle}_{\underline{a}(t')\otimes I} = \text{Prob}(\lambda)^{|\Psi(t)\rangle}_{\underline{a}(t')\otimes I}$$

But the R.H.S. of this equation is the probability of finding the result λ by measuring $\underline{a}$ on A at time t' in the absence of perturbation. Since the state-vector at t' is the same as the state-vector at t.

In other words the probability of obtaining the result λ, by measuring $\underline{a}$ on A at any time t' is independent of any possible perturbation of the system $\underline{B}$ between t and t'.

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Figure caption

Rotation of ~~vector~~ unit vector $\vec{OB}$ through angle ϕ about axis $\vec{OA}$ into new position $\vec{OB'}$. OC is the bisector of the angle between $\vec{OB}$ and $\vec{OB'}$. The plane NOC, denoted by Π, is shown shaded.